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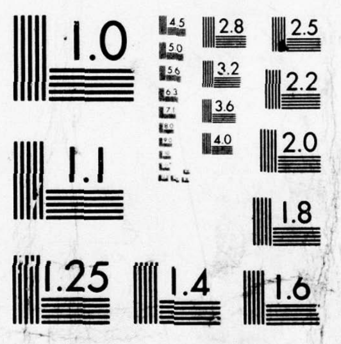

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MEAN POWER, UNIFORM (CHEBYSHEVSKIY) AND QUASI-UNIFORM APPROXIMATIONS

by

Ye. Ya. Remez

EDITED TRANSLATION

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Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

GREEK ALPHABET

Alpha	Α α ɑ		Nu	Ν ν	
Beta	Β β		Xi	Ξ ξ	
Gamma	Γ γ		Omicron	Ο ο	
Delta	Δ δ		Pi	Π π	
Epsilon	Ε ε ϵ		Rho	Ρ ρ ϱ	
Zeta	Ζ ζ		Sigma	Σ σ ς	
Eta	Η η		Tau	Τ τ	
Theta	Θ θ ϑ		Upsilon	Υ υ	
Iota	Ι ι		Phi	Φ φ ϕ	
Kappa	Κ ϰ κ ϰ		Chi	Χ χ	
Lambda	Λ λ		Psi	Ψ ψ	
Mu	Μ μ		Omega	Ω ω	

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

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MEAN POWER, UNIFORM (CHEBYSHEVSKIY) AND QUASI-UNIFORM APPROXIMATIONS*

Ye. Ya. Remez
(Presented by Acadamician M. A. Lavrent'yev 14 February 1948)

1. General conditions and designations. Let E signify a given measureable point set of measures $\mu E (0 < \mu E < \infty)$ in some abstract space (in the Fréchet meaning [8, 9]) in which a completely additive non-negative measure μe has been determined for some additive system of sets $\{e\}$. We will designate various elements (points) of set E by the letter x. Further, assume

$$v_0(x),\ v_1(x), \ldots,\ v_n(x) \tag{1}$$

will be n + 1 assigned numerical functions **, determined and

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*Some of the preceding works by the author [1, 2] proposed a method of successive quadratic approximations for the actual construction of generalized polynomials of the least mean power deviation from zero. This method, which agrees under extremely general conditions which correspond to the nature of the very problem of a mean power approximation, naturally put forth the question of the further applications for problems of the Chebyshevskiy type whose tie with problems of a mean power approximation was, however, only established for cases of continuous polynomials [3-7]. This report examines the relationships of this tie with that power of generality which corresponds to the generality of the very problem of the mean power approximation and the method which has been mentioned, in which regard, naturally (compare with [6]), here it is necessary to be satisfied with the establishment of the very convergence of the Polia-Jackson process without entering into a consideration of the speed of the convergence.

**They may be real or complex just as the coefficients c_i introduced below. In the latter case, the condition of measureability $v_i(x)$ is reduced to the measureability of the actual and imaginary parts separately.

measureable (μ) on set E, limited or unlimited but finite everywhere. Finally, we will assume these n + 1 functions metrically linearly independent in the following meaning: subset $E_1 \subseteq E$ of those points in which some generalized polynomial $\Omega(x) = \sum_{i=0}^{n} c_i v_i(x) \left(\sum_{i=0}^{n} |c_i| > 0\right)$, becomes zero always satisfies the condition

$$\mu E_1 < \mu E. \qquad (2)$$

2. Two lemmas on functions which are metrically linearly independent.

Lemma 1. Under the condition of a metric linear independence in equality (2) can always be replaced by an inequality of the type

$$\mu E_1 \leqslant g < \mu E, \qquad (3)$$

where the constant g does not depend on the selection of coefficient c_i of polynomial Ω.

Proof. Let us introduce for further designation

$$\max\{|c_0|; |c_1|, \ldots, |c_n|\} = L[\Omega] = L(c_0, c_1, \ldots, c_n). \qquad (4)$$

Proving the lemma from the opposite, let us assume that the required number g does not exist. Then, for any natural number ν the polynomial $\Omega_\nu(x) = \sum_{i=0}^{n} c_{i\nu} v_i(x)$, $L[\Omega_\nu] = 1$, which becomes zero on set E_ν, $\mu E_\nu > \mu E - 1/\nu$ would be found. Isolating the sequence of subscripts ν_m under the condition of the simultaneous existence of $\lim_{m \to \infty} c_{i\nu_m} = c_{i0}$ (i = 0, 1, ..., n) we obtain the polynomial $\Omega_0(x) = \sum_{i=0}^{n} c_{i0} v_i(x)$, $L[\Omega_0] = 1$, becomes zero in all points of set $E_0 = \overline{\lim_{m \to \infty}} E_{\nu_m}$, the measure of which (compare with [9], Chapter I) exceeds any of the numbers $\mu E - 1/\nu_m$ and, consequently, is equal to μE which contradicts the condition for linear independence of (2).

Lemma 2. Having some system of metrically linear independent functions (1), for which, in accordance with what has been proven, the inequality (3) is satisfied with some fixed value of the number g, let us examine the corresponding polynomials $\Omega(x)$ for which $L[\Omega] = 1$. Then, any ε taken within the limits of $0 < \varepsilon < \mu E - g$, can be opposed by such a $\lambda = \lambda_\varepsilon > 0$ so that the inequality $|\Omega(x)| \leqslant \eta$ could not be satisfied on any set $E_1 \subseteq E$ of the measure $\mu E_1 \geqslant g + \varepsilon$, if $\eta < \lambda$.

The proof (from the opposite) is extremely similar to the proof of the preceeding lemma [10, 2].

Corollary. With an arbitrary value of $L = L[\Omega]$ the confirmation of the lemma retains its force if we replace $\lambda = \lambda_\varepsilon$ by $L\lambda$.

3. Formulation of the three approximation problems. Isolating as allowable polynomials those for which $c_0 = 1$:

$$\Phi(x) = v_0(x) + P(x) = v_0(x) + \sum_{i=1}^{n} c_i v_i(x), \tag{5}$$

we consider the next three problems.

a) The problem of the mean power approximation in accordance with the given subscript $m > 1$:

$$\left[\frac{1}{\mu E}\int_E |\Phi(x)|^m \, d\mu\right]^{1/m} = \delta_m[\Phi] = \delta_m(c_1, \ldots, c_n) = \min; \tag{6}$$

the finiteness of the integrals [8, 9] $\int_E |v_i(x)|^m \, d\mu$ $(i = 0.1, \ldots, n)$ is assumed.

b) The problem of the quasi-uniform approximation:

$$\operatorname{vrai} \max_{x \in E} |\Phi(x)| = \delta_0^*[\Phi] = \delta_0^*(c_1, \ldots, c_n) = \min. \tag{7}$$

Here function (1) is assumed to be limited almost everywhere on set E.

c) The problem of a uniform approximation

$$\sup_{x \in E} |\Phi(x)| = \delta_0[\Phi] = \delta_0(c_1, \ldots, c_n) = \min, \tag{8}$$

where function (1) is assumed to be limited in the general meaning on set E.

4. On mean power approximations. Let us place in conformance with each $\Phi(x)$ a "point" $(c_1, c_2, \ldots, c_n)$ in Cartesian space (Menger-Fréchet terminalogy) of the corresponding number of measurements. From lemma 1 and the corollary of lemma 2 it is easy to see (compare the proof of theorem 1 below) that $\delta_m[\Phi] \to \infty$ with $L[\Phi] \to \infty$. Thus, we come to the problem of finding the minimum of the continuous (on the strength of the Minkovskiy inequality) function $\delta_m(c_1, \ldots, c_n)$ in a limited and closed region of the mentioned Cartesian space and the existence of at least one solution $\Phi_m = v_0 + c_{1m} v_1 + \ldots + c_{nm} v_n$ proves to be guaranteed in accordance with the Bolzano-Weierstrass theorem. The uniqueness of the solution of $\Phi(x)$ is easily established further by the corresponding extention of the method of proof employed by Jackson [4].

Theorem 1. If at least one of the admissible polynomials $\bar{\Phi} = v_0 + \bar{c}_1 v_1 + \ldots + \bar{c}_n v_n$ is limited almost everywhere on set E, in which regard we assume the intergrals $\int_E |v_i(x)|^m d\mu \ (i = 0, 1, \ldots, n)$ here finite for all $m > 1$, then the coefficients of the polynomial-solution of $\Phi_m(x)$ is uniformly limited for all $m > 1$.

The proof is obtained from the inequality $\delta_m[\Phi_m] \leqslant \delta_m[\bar{\Phi}]$ on the basis of the corollary of lemma 2: taking the fixed $\varepsilon > 0$ $(\varepsilon < \mu E - g)$ and assuming $L[\Phi_m] = L_m$, $\operatorname{vrai\,max} |\bar{\Phi}(x)| = M$, we have $(L_m \lambda_\varepsilon)^m (\mu E - g - \varepsilon) <$ $< M^m \cdot \mu E$, whence $L_m < \frac{M \cdot \mu E}{\lambda_\varepsilon (\mu E - g - \varepsilon)}$ for all $m > 1$.

5. Quasi-uniform approximations and their connection with the mean power approximations: the first generalization of the Polia-Jackson theorem.

Considering the continuity of the function $\delta_0^*(c_1, \ldots, c_n)$ which can be established without difficulty and, as in the preceeding heading, relying on the corollary of lemma 2, we are convinced similarly of the existence of at least one solution $\Phi_0^* = v_0 + c_{10}^* v_1 + \ldots + c_{n0}^* v_n$ to problem (7). Here, the solution can be single-valued or infinite-multiply-valued: together with Φ_{01}^*, Φ_{011}^*, $\Phi_{01}^*(x)\cos^2\varphi + \Phi_{011}^*(x)\sin^2\varphi$ $(0 \leqslant \varphi = \text{const} \leqslant \pi/2)$ will also be a solution.

Theorem 2. Under conditions of problem (7) which is being considered the following limiting relationship always occurs

$$\lim_{m \to \infty} \delta_0^*[\Phi_m] = \delta_0^*[\Phi_0^*]. \tag{9}$$

Proof. On the basis of theorem 1 and considering, in addition, the continuous dependence of the value $\delta_0^*[\Phi]$ on the coefficients of the polynomial $\Phi(x)$, it obviously will be sufficient to be convinced that any polynomial $\bar{\Phi} = v_0 + \bar{c}_1 v_1 + \ldots + \bar{c}_n v_n$, is "limiting" (in the sense of $c_{im} \to \bar{c}_i$, i + 1, 2, ..., n) for any sequence Φ_{m_ν}, where $m_\nu \to \infty$ with $\nu \to \infty$, is one of the polynomials $\Phi_0^*(x)$. But if, assuming the opposite, we assume that $\delta_0^*[\bar{\Phi}]$ exceeds the value $\delta_0^*[\Phi_0^*] = \rho$ by more than $3\varepsilon > 0$, then $|\bar{\Phi}(x)|$ will exceed $\rho^* + 2\varepsilon$ on some set $E_1 \subseteq E$ of the measure $\mu E_1 = \sigma > 0$; almost everywhere on the same set we will have $|\Phi_{m_\nu}(x)| > \rho^* + \varepsilon$ with $\nu > \nu_\varepsilon$; this, as can be easily seen, leads to the explicit contradiction with the equality $\delta_{m_\nu}[\Phi_{m_\nu}] \leqslant \delta_{m_\nu}[\Phi_0^*]$ with sufficiently large m_ν.

6. Uniform approximations and the corresponding second generalization of the Polia-Jackson theorem.

In the case of problem (8) the proof of the existence of at least one solution $\Phi_0 = v_0 + c_{10} v_1 + \ldots + c_{n0} v_n$ is accomplished in an absolutely similar manner to the case of the preceding problem (7), in this case, too, the solution proving to be either single-

valued or infinite-multiple-valued.* As regards the generalization of the Polia-Jackson relationship, here it proves possible, in general, only in the form of the next assumption which is a direct corollary of theorem 2 which we have proved.

Theorem 2'. The maximum relationship

$$\lim_{m \to \infty} \delta_0 [\Phi_m] = \delta_0 [\Phi_0] \tag{10}$$

occurs everytime that functions (1) and set E which are subordinate to the general requirements of headings 1 and 3 (c) are determined in such a way that for polynomials $\Phi(x)$ the following condition is satisfied

$$\sup_{x \in E} |\Phi(x)| = \operatorname{vrai}\max_{x \in E} |\Phi(x)|. \tag{11}$$

On the strength of this theorem the limiting polynomial of any converging sequence of polynomials $\Phi_{m_\nu}(x)$ $(m_\nu \to \infty)$ proves to be one of the Chebyshovskiy (generalized) polynomials $\Phi_0(x)$ with the satisfaction of condition (11).

If, for example, we realize the abstract set E in the form of some Euclid region R_n, then for the satisfaction of condition (11) it will be sufficient to require that the polynomials $\Phi(x)$ possess at each point $x \in E$, so to say, a partial continuity (attenuated form of an asymptotic continuity), namely - a continuity relative to some subset which has the measure > 0 in any point in the vicinity. In particular, if, for example, $E \equiv (a, b)$ (open

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*Since the matter concerns the problem (8) itself outside its tie with the two other problems (6) and (7), neither set E nor the determination of measures μe, as it is not difficult to understand, play any substantial role in it: only the set of systems of values of functions (1) with $x \in E$ actually plays a role. Exclusion of the "parameter" x leads to the "problem of minimum approximation," which pertains to the solution of the system (usually innumerably infinite) of incompatible linear (n + 1)-term equations which was a subject of several of the author's studies [11].

interval in R_1), then it proves to be more than sufficient to assume the one-sided (let us say, right-sided) continuity of all n + 1 functions $v_i(x)$ at each point.

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